Option-based Spot Volatility Index (SV)

1 Overview

Option-implied volatility measures are widely used for risk management by investors and regulators. Two of the most popular are the Black-Scholes implied volatility (BSIV) and the CBOE volatility index (VIX). However, both BSIV and VIX suffer from a non-trivial upward bias in measuring the underlying spot volatility due to time variation in both volatility and jump risk as well as the compensation demanded by investors for bearing these risks (represented by the wedge between the statistical and risk-neutral distribution). In practice, accounting for the correct value of spot volatility is important for many risk management and investment decisions. The **option-based spot volatility index (SV)**, developed here, is intended to provide users with a jump-robust estimator of spot volatility.

The construction of the SV index is based on the theoretical work of Todorov (2019). Specifically, following Carr and Madan (2001), Todorov (2019) "spans" the risk-neutral conditional characteristic function of the price increment via a portfolio of short-dated out-of-money (OTM) options and evaluates it at a high value of the characteristic exponent to minimize the impact of jumps. The estimation is based on the following approximation

$$\mathcal{L}_{t,T}(u) \;=\; \mathbb{E}_t^Q\big(e^{iu(x_{t+T}-x_t)/\sqrt{T}}\big) \;\approx\; e^{-u^2\sigma_t^2/2}, \quad \text{for } u>0 \text{ and } T\approx 0,$$

where $\mathbb{E}_t^Q$ stands for the expectation under the risk-neutral probability, x_t is the logarithm of the underlying stock price, and σ_t^2 is the spot (diffusive) variance at time t. The option portfolio behind the spot volatility index weighs near-the-money options heavily, while the weights generally are smaller and declining for strike prices further out-of-the-money.

We provide daily option-based spot volatility index (SV) estimates for the S&P 500 Index, starting from 2008. The SV estimates for 2008-2010 are calculated from regular S&P 500 Index options and starting from 2011 on from S&P 500 weekly options.

2 Calculation

2.1 The SV Formula

The option-based spot volatility index (SV) is constructed based on the near-term and next-term options with time-to-maturity T_{near} and T_{next}:

$$\text{SV}_t \;=\; 100 \times \sqrt{\frac{V_{t,T_{\text{near}}} + V_{t,T_{\text{next}}}}{2}} \tag{1}$$

where

$$V_{t,T_{\text{near}}} = \text{ spot variance estimator based on near-term options,}$$
$$V_{t,T_{\text{next}}} = \text{ spot variance estimator based on next-term options.}$$

The option-based spot variance estimator at time t with time-to-maturity T is given by:

$$\mathrm{V}_{t,T} = -\frac{2}{T\,\widehat{u}_{t,T}^2}\,\Re\left\{\log(\mathcal{L}_{t,T}(\widehat{u}_{t,T}))\right\}, \tag{2}$$

$$\mathcal{L}_{t,T}(u) = 1-(u^2+iu)\sum_{j=1}^{N_{t,T}} e^{iu\,[\log(K_{j-1})-\log(F_{t,T})]}\,\frac{O_{t,T}(K_{j-1})}{K_{j-1}^2}\,\Delta_j, \quad u\in\mathbb{R}, \tag{3}$$

where

$$\begin{aligned}
&N_{t,T} = \text{ number of OTM option strikes with maturity } T,\\
&F_{t,T} = \text{ forward price with maturity } T,\\
&K_j = \text{ strike price},\\
&O_{t,T}(K_j) = \text{ option price for strike } K_j,\\
&\Delta_j = \text{ strike increment, } K_j - K_{j-1},\\
&T = \text{ time-to-maturity (annualized)},
\end{aligned}$$

and $i=\sqrt{-1}$ is the imaginary unit and $\Re$ denotes the real part of a complex number. We set $\widehat{u}_{t,T} = \widehat{u}_{t,T}^{(1)} \wedge \widehat{u}_{t,T}^{(2)}$ for,

$$\begin{aligned}
\widehat{u}_{t,T}^{(1)} &= \inf\left\{u\geq 0:\,|\widehat{\mathcal{L}}_{t,T}(u)|\leq 0.2\right\},\\
\widehat{u}_{t,T}^{(2)} &= \operatorname{argmin}_{u\in[0,\overline{u}]}|\widehat{\mathcal{L}}_{t,T}(u)|, \quad \text{with} \quad \overline{u}_{t,T} = \sqrt{\frac{2}{T}\,\frac{\log(1/0.05)}{\widehat{\mathrm{BSIV}}_{t,T}^2}},
\end{aligned}$$

where $\widehat{\mathrm{BSIV}}_{t,T} =$ Black-Scholes Implied Volatility for the Strike closest to the Forward.

Figure 1 illustrates the value decomposition of $\mathcal{L}_{t,T}(\widehat{u}_{t,T})$, from which the option-based spot variance estimator is constructed. The example uses end-of-week options written on the S&P 500 Index (SPXW) with 4 business days to expiration. The value of the real part of $\mathcal{L}_{t,T}(\widehat{u}_{t,T})$ is determined by a long-short portfolio of near-the-money call and put options, while the contribution from deep OTM options (strikes more than 2 standard deviations from the current stock price) is almost zero. Since the option prices around the money are nearly symmetric, the value of the portfolio corresponding to the imaginary part of $\mathcal{L}_{t,T}(\widehat{u}_{t,T})$ is close to zero. We note that the construction of the CBOE VIX Index is based on a similar spanning of the "log-function," but the weights in that OTM option portfolio are the reciprocal of the squared strike prices. As a result, deep OTM options contribute much more significantly to the value of the VIX index.

2.2 Data

We obtain short-dated option data (end-of-day, best bid and ask) for S&P 500 Index and the 30-day zero-coupon bond rate from the OptionMetrics IvyDB US file from 2007 to 2017. We apply the following filters to the raw option data:

- Drop missing and zero bid and ask records;
- Drop the bid and ask records with a ratio of ask to bid higher than 10;
- Drop (t,T) pairs that have a lowest available option price above \$0.5;
- Drop (t,T) pairs with less than 3 distinct OTM calls and puts;
- Retain up to 2 short-dated options per day, with $T\geq 2$.

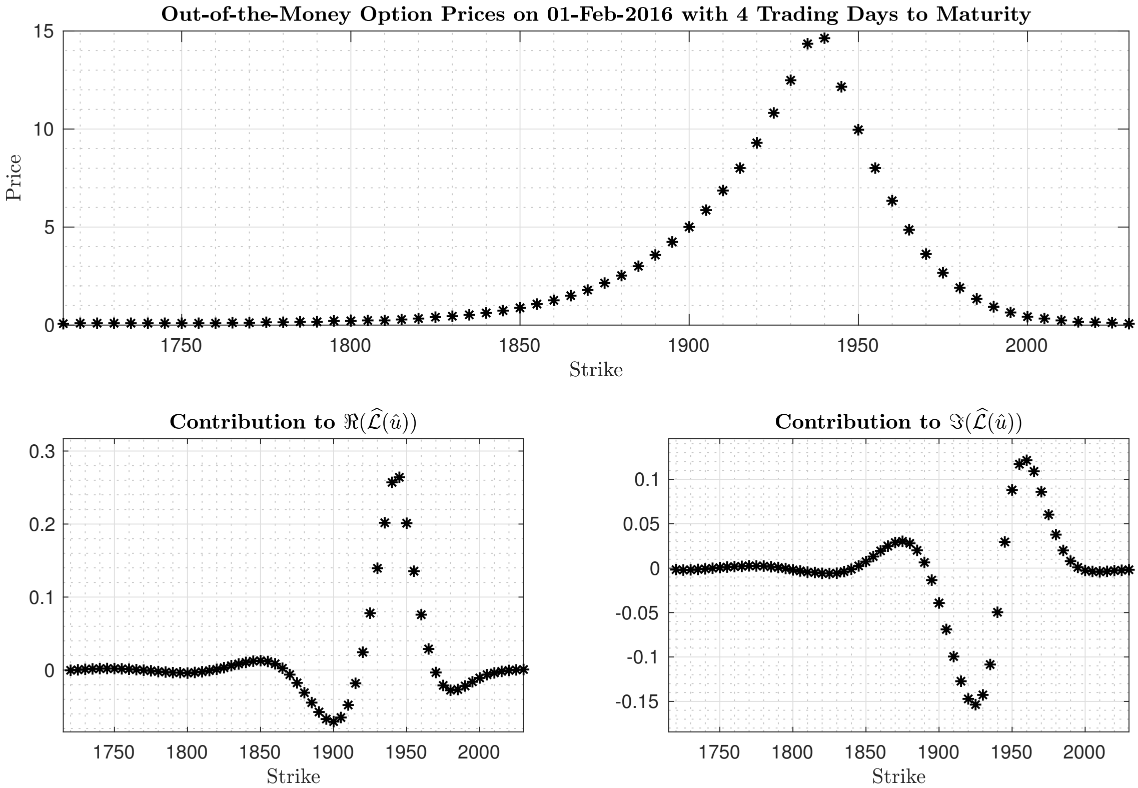

Figure 1: **Decomposition of $\mathcal{L}_{t,T}(u)$.** The upper panel plots short-maturity OTM option prices as a function of their strike. The lower panel plots their contributions to $\mathcal{L}_{t,T}(\widehat{u}_{t,T})$. The forward price is 1938.

2.3 Determine Time-to-maturity

We adopt a business time convention. For AM-settled options (e.g. the traditional monthly SPX option), we adjust the time-to-maturity with an overnight factor calculated from the ratio of average volatility during overnight and intraday periods based on 5-minute returns on the CME E-mini S&P 500 Index futures.

2.4 Select Out-of-money Options

We select the OTM options based on the synthetic forward level computed from put-call parity via,

$$F_{t,T} = K_{near,T} + e^{r_f \tau} \left[C_{t,T}(K_{near,T}) - P_{t,T}(K_{near,T}) \right], \tag{4}$$

where

$$r_f = \text{Risk-free Rate},$$
$$K_{near,T} = \text{Strike Price with Smallest Put-Call Absolute Price Difference.}$$

We use the 3 near-the-money put-call pairs with minimum absolute put-call price difference to compute the forward rate and take the median of these estimates. The ATM option K_{ATM}, is the call or put option with strike equal or immediately below the forward price $F_{t,T}$. If there are both call and put options with strikes of K_{ATM}, we select the put option.

2.5 Interpolate Options for Missing Strikes

In practice, available option price quotes have uneven strike price grid. In order to minimize the approximation error in computing $\mathcal{L}_{t,T}(u)$ resulting from this, we fill-in strike price gaps by linearly interpolating the Black-Scholes implied volatilities calculated from the observed option prices on a uniform strike price grid with $\Delta K = \$5$:

$$\mathbf{K} = [K_{low} : \Delta K : K_{high}] \tag{5}$$

where

$$\begin{aligned} K_{low} &= \text{ Lowest observed Strike Price,} \\ K_{high} &= \text{ Highest observed Strike Price.} \end{aligned}$$

3 Code and Technical Notes

We provide MATLAB codes for the calculation of the SV index and a sample of simulated option data based on the Monte Carlo Study in Todorov (2019). The MATLAB codes and sample data are available to download on the website. **SVDemo.m** illustrates the construction of the SV index in Equation (1) and the usage of **GetV.m**, where **GetV.m** is the main function to calculate the option-based spot variance in Equation (2):

function V = GetV(options,T,spot)

- Input:
  - options: a data matrix with columns being strike price (K), option prices (O), and Black-Scholes implied volatility (IV), sorted by strike price K in ascending order.
  - T: time-to-maturity in year unit
  - spot: spot/forward price
- Output:
  - V: annualized option-implied spot variance

References

Carr, P. and Madan, D., "Optimal Positioning in Derivatives Securities," *Quantitative Finance* 1, 19-37, 2001.

Todorov, V., "Nonparametric Spot Volatility from Options," *Annals of Applied Probability*, forthcoming 2019.